Dissertation Abstract

A Numerical Simulation Study of Data-driven Pole Placement

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1 Abstract

This dissertation is concerned with numerical simulation studies on the state-feedback data-driven pole placement method. The data-driven pole placement method can precisely identify the state space model and pole placement gain simultaneously from a set of measurement data of the linear time-invariant system under certain conditions. In this study, solutions of several difficulties of the method for practical applications are investigated by numerical simulations.

First, the data-driven pole placement method is applied to a self-balancing robot which is a nonlinear system. By numerical simulations with nonlinear differential equation of the self-balancing robot, it is shown that the linearized model can be identified for the noisy case where the measurement noise exists together with noiseless cases. In particular, it is revealed that the suitable linearized model and pole placement gain can be identified by using the data sufficiently near the equilibrium.

Second, it is shown that the total least square and a prefilter are effective to the data-driven pole placement method when the measurement data is contaminated by noise. It is also shown that the random exciting signal is more suitable than the chirp exciting signal.

Finally, the data-driven pole placement method is extended to online tuning, real-time updating the closed loop system. Its capability is also investigated by numerical simulations of the self-balancing robot. It is shown that the method can update the state space model of the self-balancing robot and the pole placement gain for noisy measurement.

2 Data-driven pole placement

Pole placement, also called pole assignment or eigenvalue assignment, is a standard controller synthesis method in which the locations of the closed-loop poles can be determined by setting a controller gain. The eigenvalues of the system correspond to the pole locations and they affect the system response such as stability, convergence rate, disturbance rejection and noise immunity. For stability issue, the poles of the system should be inside the unit circle in the discrete time system or should be the left-half plane in the continuous time system. Pole placement method works on setting the desired pole location and then moving the poles of the system to these desired pole locations by using the feedback gain to specify the desired system response. For pole placement control design, all state variables are assumed to be measurable and available for feedback and, the system is assumed to be completely controllable. Various pole placement methods have broadly been developed.

In contrast to the standard pole placement approach that assumes the state-space model is known and given, a different pole placement approach that does not use such assumptions has recently been proposed. A salient feature of the approach is that from a pair of state and input measurement we can simultaneously obtain the state-space model and the pole placement gain. The basic principle of this approach is based on unfalsified control, which is also known as data-driven control.

Data-driven pole placement was proposed for the state feedback control of discrete-time linear systems. Various control methods for the pole placement problem have well-known for a long time. In state feedback pole placement problem, the state feedback gain must be determined for a given system such that the closed-loop poles coincide with the desired locations. This is also a well-known problem, and various pole placement methods have been extensively discussed in many works of literature [1, 2, 3, 15].

In standard pole placement methods, a state space model is assumed to be given by a system identification technique using data from past experiments. Whereas the traditional approach combines the identification of the state space model with the standard pole placement method, an alternative approach called "data-driven pole placement" has recently been proposed [5]. In this approach, the state space model and pole placement feedback gain are identified simultaneously from the set of state measurements and control input sequences. The method proposed in [5] is based on the data-driven control framework ([17] and references therein) such as unfalsified control [6], virtual reference feedback tuning (VRFT) [18, 19], or fictitious reference iterative tuning (FRIT) [8, 20, 21, 22].

Consider the discrete-time linear time-invariant system and a static state feedback

$$x(k+1) = Ax(k) + Bu(k), \tag{1}$$

$$u(k) = Fx(k) + v(k), \tag{2}$$

where $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$, $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^m$ is the input vector, $v \in \mathbf{R}^m$ is the external input to the closed loop system, and $F \in \mathbf{R}^{m\times n}$ is the feedback gain.

The data-driven pole placement problem was formulated in [5] as follows.

Problem 1 *We assume that the order of the plant n is known, pair (A, B) is controllable but the exact value is unknown, and B is of full rank. Let* $\Lambda = \{p_1, \ldots, p_n\}$ *be a self-conjugate set of n complex numbers in the unit circle. Given the input and output measurement data sequence* $(x_0(k), u_0(k))$ *of (1), find a state feedback gain F from the observed data* $(x_0(k), u_0(k))$ *such that* $\{\lambda_i(A + BF)\} = \Lambda$.

In a conventional approach, this problem is solved in two steps: A and B are identified from $x_0(k), u_0(k)$, then F is derived using the standard pole placement algorithms. In contrast, the data-driven pole placement method solves the two steps simultaneously. To achieve this, the method uses the equivalency between the closed-loop system

$$x(k+1) = (A + BF)x(k) + Bv(k) \tag{3}$$

with the desired pole placement gain F and

$$x_{\mathrm{d}}(k+1) = A_{\mathrm{d}}x_{\mathrm{d}}(k) + B_{\mathrm{d}}v(k), \tag{4}$$

$$x_{\mathrm{d}}(k) = Tx(k), \tag{5}$$

where $(A_\mathrm{d}, B_\mathrm{d})$ with $\lambda_i(A_\mathrm{d}) = p_i$ is an appropriate controllable pair. This equivalency requires the nonsingular matrix T to exist. We remove v from (4) by using (2), to obtain

$$x_\mathrm{d}(k+1) = A_\mathrm{d}x_\mathrm{d}(k) + B_\mathrm{d}u(k) - B_\mathrm{d}Fx(k). \tag{6}$$

Then, using (5), we obtain

$$Tx(k+1) = A_\mathrm{d}Tx(k) + B_\mathrm{d}u(k) - B_\mathrm{d}Fx(k). \tag{7}$$

If $(x_0(k), u_0(k))$ $(k = i, \dots, i+N)$ satisfies (7),

$$TX_0P_1 = A_\mathrm{d}TX_0P_2 + B_\mathrm{d}U_0 - B_\mathrm{d}FX_0, \tag{8}$$

where

$$X_0 = \begin{bmatrix} x_0(i) & x_0(i+2) & \cdots & x_0(i+N) \end{bmatrix}, \tag{9}$$

$$U_0 = \begin{bmatrix} u_0(i) & u_0(i+1) & \cdots & u_0(i+N-1) \end{bmatrix}, \tag{10}$$

$$P_1 = \begin{bmatrix} 0_{1\times N} \\ I_N \end{bmatrix}, \quad P_2 = \begin{bmatrix} I_N \\ 0_{1\times N} \end{bmatrix}. \tag{11}$$

In [5], (8) is cast into

$$S_1 \begin{bmatrix} T \\ F \end{bmatrix} X_0P_1 + S_2 \begin{bmatrix} T \\ F \end{bmatrix} X_0P_2 = B_\mathrm{d}U_0 \tag{12}$$

$$S_1 = \begin{bmatrix} I_n & 0_{n\times m} \end{bmatrix}, \; S_2 = \begin{bmatrix} -A_\mathrm{d} & B_\mathrm{d} \end{bmatrix} \tag{13}$$

and

$$F = \begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix} \in \mathbf{R}^{m\times n}, \quad T = \begin{bmatrix} t_1 \\ \vdots \\ t_n \end{bmatrix} \in \mathbf{R}^{n\times n}. \tag{14}$$

The system (4) can be interpreted as a reference model within VRFT (e.g., [18, 19]) and FRIT (e.g., [8, 20, 21, 22]). The idea of eliminating v in (6) is also based on FRIT. In [8, 21, 22], a similar state feedback control problem has been discussed within the FRIT framework. To apply these FRIT techniques to the data-driven pole placement problem, the desired transfer function must be specified from u to x, rather than x_d. When precise values for (A, B) are not available, it becomes impossible to specify the zeros of the desired transfer function.

To obtain the datasets (9) by applying state feedback (2) to the system (1), the initial feedback gain F should be based on (A, B). Hence, in Problem 1, the exact value of (A, B) is assumed to be unknown.

When applying the property of Kronecker product $\mathrm{vec}(MDN) = (N^\top \otimes M)\mathrm{vec}D$ (see for example Th.2.13 in [28]) to the transpose of (12) to solve (12) for F and T, a further linear equation is derived, as follows:

$$\mathcal{X}\eta = \mathcal{U}, \tag{15}$$

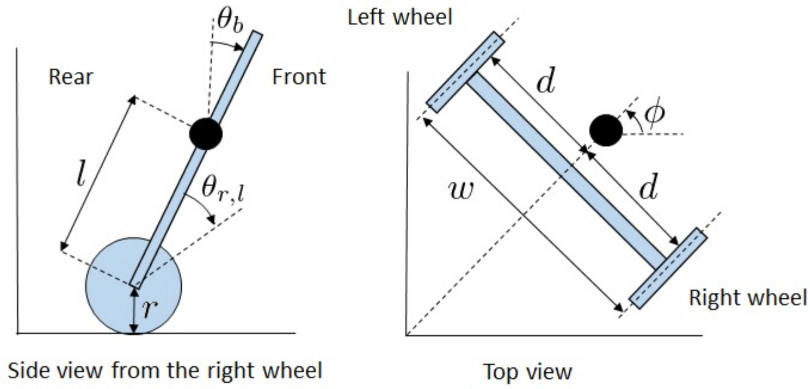

Figure 1: Coordinates of the self-balancing robot.

where

$$\eta = \begin{bmatrix} t_1 & \cdots & t_n & f_1 & \cdots & f_m \end{bmatrix}^\top \in \mathbf{R}^{(n+m)n} \tag{16}$$

$$\mathcal{X} = S_1 \otimes (X_0 P_1)^\top + S_2 \otimes (X_0 P_2)^\top \in \mathbf{R}^{nN \times (n+m)n}, \tag{17}$$

$$\mathcal{U} = \left(B_\mathrm{d} \otimes U_0^\mathrm{T}\right)(\mathrm{vec}\, I_m) \in \mathbf{R}^{nN}. \tag{18}$$

If T is nonsingular, the model coefficients can be obtained

$$A = T^{-1} A_\mathrm{d} T - T^{-1} B_\mathrm{d} F, \quad B = T^{-1} B_\mathrm{d}. \tag{19}$$

2.1 Main Numerical Simulation Results

We applied the data-driven pole placement method to the model of a 3D self-balancing robot [9, 27] shown in Fig. 1.

We have shown the simulation results to see how noise takes effects on the performance of data-driven pole placement method in [14, 16]. Although total least square (TLS) method was declared as an effective method in [5], we can see that dealing with noise in that method is still open. As every measurement of any physical quantity becomes uncertain because of it, we design FIR prefilter to deal with it effectively. Then, we apply the least square and total least square in order to get the best fit data together with the random exciting signal. We compare the results before and after applying the designed prefilter by numerical results and simulations. Then, to evaluate the response when we apply the different exciting signal, we also introduce the charp exciting signal and compare the results.

We finally compared the pole locations obtained, as shown in Fig. 2. As can be seen, a better performance was achieved when using the random exciting signal.

3 Summary

In this study, we evaluated different approaches to reducing the effect of measurement noise in data-driven pole placement methods for deriving a state space model and pole placement state feedback. Using numerical simulations of a self-balancing

Table 1: Comparison of errors.

	(initial)	(a)	(b)	(c)	(d)	(e)
noise	-	noiseless	noisy	noisy	noisy	noisy
method	-	LS	LS	TLS	TLS+PF	TLS+PF
exciting sig.	-	Random	Random	Random	Random	Chirp
$\delta\lambda(A_{\mathrm{d}1})$	0.2426	0.0007	0.4597	0.1367	**0.0466**	1.2530
ΔA_1	0.5317	0.0016	36.295	1.6678	1.8763	17.246
ΔB_1	0.0025	0.0000	0.3400	0.0481	0.0415	0.2932
$\delta\lambda(A_1)$	0.0511	0.0000	0.3920	0.0194	**0.0177**	0.4695
ΔG_1	42.333	0.0082	629.67	44.718	**29.324**	106.04
$\delta\lambda(A_{\mathrm{d}2})$	0.0029	0.0000	0.0092	0.0024	**0.0007**	0.0017
ΔA_2	0.0001	0.0000	0.0288	0.0064	0.0005	0.0007
ΔB_2	0.0004	0.0000	0.0031	0.0002	0.0002	0.0002
$\delta\lambda(A_2)$	0.0001	0.0000	0.0090	0.0012	0.0004	0.0001
ΔG_2	0.0036	0.0002	0.0525	0.0073	0.0019	0.0019

robot, which is a nonlinear system, we demonstrated the important role that prefiltering can play in reducing the interference caused by noise. Again using numerical simulation, we compared the use of two exciting signals: a random signal and a chirp signal. The use of a random exciting signal was found to be more effective with our proposed method. Further developments are needed in the methods used to cope with noise. A method such as that used in [19] may be appropriate for use in practical applications where noise is present, and adaptive control based on real-time updating [16] is a future promising approach.

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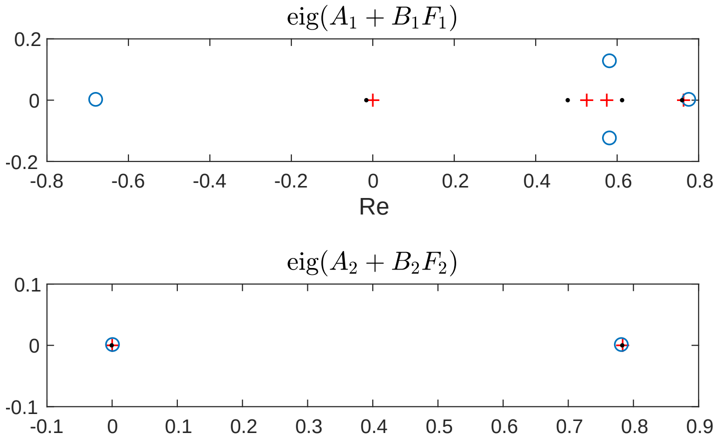

Figure 2: Comparison of pole locations ('+' indicates the desired poles, '·' those obtained by the random exciting signal and '∘' those obtained by the chirp exciting signal).

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